ESTIMATION ECOLOGICAL SUSTAINABILITY INDEX ON THE BASE OF FUZZY LOGIC

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For calculate of the Ecological Sustainability Index are using by values of the many indicators, where the main are the indicators sustainability of air quality, water quality, land quality and quality of biodiversity of the considered region. And indicators of air and water qualities dominate over the others owing to the objective reasons. In other words, from the estimations levels of sustainability of this indicator formalize the general estimation of an “ecology sustainability” category. Obviously, that the estimation of ecology sustainability are the multi-criterial procedures meaning application of a composite rule of aggregate estimation of each region on the basis of accessible information about the qualities of air, water, land and biodiversity. As a rule, these data are semistructured and that is why for their adequate represent more advisable use fuzzy numbers.

For assessment the indicators of sustainability of air quality, water quality, land quality and biodiversity we choose the five level of composition: \( u_1\) – low; \( u_2\) – below the average; \( u_3\) – average; \( u_4\) – above the average; \( u_5\) – high. Better to say, under the set \( C=(u_1,u_2,u_3,u_4,u_5)\) we will understand set of signs on which levels of sustainability of the indicators are classified. Then, suppose criteria of assessment by fuzzy sets, the procedure of multi-criterial assessment of ecological sustainability we realize with use of sufficient set of fuzzy implicate rules in form as “If..., then...”, and on that basis we establish for it the corresponding gradation scale. Further, on the given scale we realize a quantities estimation of ecological sustainability of region.

For establish levels of the ecological sustainability we will used by next set of consistent implicate rules, which take into account indicators of air and water quality over the indicators of land quality and biodiversity:

- \( e_1\): if air quality sustainable comprehensible and water quality are standard, then the level of ecological sustainability is satisfactory (meets the minimum standard requirements);
- \( e_2\): if in addition to the above-stated requirements biodiversity is sustainable does not vary, then the level of ecological sustainability is more than satisfactory;
- \( e_3\): if additionally to conditions \( e_2\) the land quality saves sustainable state, then the level of ecological sustainability is perfect;
- \( e_4\): if in considered region take place stipulated in \( e_3\) except a biodiversity, then the level of ecological sustainability is very satisfying;
- \( e_5\): if the air quality sustainable comprehensible, water quality standard, land quality saves sustainable state, however biodiversity is not sustainable, then the level of ecological sustainability of the region nevertheless will be satisfactory;
- \( e_6\): if the air quality is not sustainable and land quality does not save sustainable state, then the level of ecological sustainability of region unsatisfactory.

The analysis of the resulted statements allow to reveal consider to offered model four exogenous criteria using to the estimation of ecology sustainability: \( X_1\) – sustainable quality of air consist; \( X_2\) – sustainable quality of water consist; \( X_3\) – sustainable quality of land consist; \( X_4\)– sustainable of biodiversity, and the endogen sign \( Y\) – sufficiency of the level of ecological sustainability. Then having defined the considered valued (fuzzy terms) of linguistic variables \( X_i\) (\( i=1\div4\)) and \( Y\), on the basis of the resulted statements we will construct fuzzy implicative rules in the form:

- \( e_1\): if \( X_1=\text{SUSTAINABLE COMPREHENSIVE}\) and \( X_2=\text{SUSTAINABLE STANDARD}\), then \( Y=\text{SATISFACTORY}\);
- \( e_2\): if \( X_1=\text{SUSTAINABLE COMPREHENSIVE}\) and \( X_2=\text{SUSTAINABLE STANDARD}\) and \( X_4=\text{SUSTAINABLE INVARIABLE}\), then \( Y=\text{MORE THAN SATISFACTORY}\);

If $X_1=\text{SUSTAINABLE COMPREHENSIVE}$ and $X_2=\text{SUSTAINABLE STANDARD}$ and $X_3=\text{SUSTAINABLE DOES NOT VARY (CHANGE)}$ and $X_4=\text{SUSTAINABLE CONSTANT}$, then $Y=\text{IRREPROACHABLE}$;

If $X_1=\text{SUSTAINABLE COMPREHENSIVE}$ and $X_2=\text{SUSTAINABLE STANDARD}$ and $X_3=\text{SUSTAINABLE DOES NOT VARY (CHANGE)}$, then $Y=\text{VERY SATISFACTORY}$;

If $X_1=\text{SUSTAINABLE COMPREHENSIVE}$ and $X_2=\text{SUSTAINABLE STANDARD}$ and $X_3=\text{SUSTAINABLE DOES NOT VARY (CHANGE)}$ and $X_4=\text{UNSUSTAINABLE}$, then $Y=\text{SATISFACTORY}$;

If $X_1=\text{UNSUSTAINABLE}$ and $X_3=WORSENS$, then $Y=\text{UNSATISFACTORY}$.

Further, linguistic variable $Y$ we will set on discrete set $J=(0;0.1;0.2;..;1)$. Then using in implicative rules its values – fuzzy terms we can write with the help of the next membership functions \[1\]:

$\tilde{S} = \text{COMPREHENSIVE} (\mu_{\tilde{S}}(x) = x, x \in J);$
$M\tilde{S} = \text{MORE THAN COMPREHENSIVE} (\mu_{M\tilde{S}}(x) = x^2, x \in J);$
$\tilde{V}\tilde{S} = \text{VERY COMPREHENSIVE as: } \mu_{\tilde{V}\tilde{S}}(x) = 1-x, x \in J).$

Fuzzifications of terms in the left part accepted rules we will realize with the help of Gaussian membership functions \[1\] restoring fuzzy sets on base vector $(0, 25, 50, 75, 100)$ (fig.1) and values for \(\sigma_k\) steal up from degree of important of an indicator.

Thus, criteria estimation of sustainability indicators we will define in the form of following fuzzy sets:

- **SUSTAINABLE COMPREHENSIVE:** $\tilde{A} = \frac{0.000015}{u_1} + \frac{0.00193}{u_2} + \frac{0.0622}{u_3} + \frac{0.4994}{u_4} + \frac{1}{u_5};$
- **SUSTAINABLE STANDARD:** $\tilde{B} = \frac{0.000015}{u_1} + \frac{0.00193}{u_2} + \frac{0.0622}{u_3} + \frac{0.4994}{u_4} + \frac{1}{u_5};$
- **SUSTAINABLE DOES NOT VARY:** $\tilde{C} = \frac{0.00285}{u_1} + \frac{0.010134}{u_2} + \frac{0.1299}{u_3} + \frac{0.6}{u_4} + \frac{1}{u_5};$
- **SUSTAINABLE CONSTANT:** $\tilde{D} = \frac{0.00193}{u_1} + \frac{0.02973}{u_2} + \frac{0.2096}{u_3} + \frac{0.676634}{u_4} + \frac{1}{u_5};$

Then with the account of these formalisms we formalize fuzzy rules as:

- $e_1$: If $X_1=\tilde{A}$ and $X_2=\tilde{B}$, then $Y=\tilde{S}$;
- $e_2$: If $X_1=\tilde{A}$ and $X_2=\tilde{B}$ and $X_3=\tilde{D}$, then $Y=M\tilde{S}$;
- $e_3$: If $X_1=\tilde{A}$ and $X_2=\tilde{B}$ and $X_3=\tilde{C}$ and $X_4=\tilde{D}$, then $Y=\tilde{P}$;
- $e_4$: If $X_1=\tilde{A}$ and $X_2=\tilde{B}$ and $X_3=\tilde{C}$, then $Y=V\tilde{S}$;
- $e_5$: If $X_1=\tilde{A}$ and $X_2=\tilde{B}$ and $X_3=\tilde{C}$ and $X_4=\neg\tilde{D}$, then $Y=\tilde{S}$;
- $e_6$: If $X_1=\neg\tilde{A}$ and $X_3=\neg\tilde{C}$, then $Y=U\tilde{S}$.

Further, having calculated membership functions $\mu_{\tilde{S}_i}(u)(i=1\div5)$ for left parts of these rules we will obtain in results:
Further, we find a dot estimation of sufficiency of ecological sustainability first region as:

Let's calculate its sets level \( E_{ja} \) and corresponding capacities \( M(E_{ja}) \) by formulas

\[
M(E_{ja}) = \sum_{i=1}^{n} x_i / n
\]

- for \( 0 < \alpha < 0.000285 \): \( \Delta \alpha = 0.000285, E_{ja} = \{0; 0.1; 0.2; 0.3; 0.4; 0.5; 0.6; 0.7; 0.8; 0.9; 1\}, M(E_{ja}) = 0.50; \)
- for \( 0.000285 < \alpha < 0.1000285 \): \( \Delta \alpha = 0.1, E_{ja} = \{0; 0.1; 0.2; 0.3; 0.4; 0.5; 0.6; 0.7; 0.8; 0.9\}, M(E_{ja}) = 0.45; \)
- for \( 0.1000285 < \alpha < 0.2000285 \): \( \Delta \alpha = 0.1, E_{ja} = \{0; 0.1; 0.2; 0.3; 0.4; 0.5; 0.6; 0.7; 0.8; 0.9\}, M(E_{ja}) = 0.4; \)
- for \( 0.2000285 < \alpha < 0.3000285 \): \( \Delta \alpha = 0.1, E_{ja} = \{0; 0.1; 0.2; 0.3; 0.4; 0.5; 0.6; 0.7; 0.8; 0.9\}, M(E_{ja}) = 0.35; \)
- for \( 0.3000285 < \alpha < 0.4000285 \): \( \Delta \alpha = 0.1, E_{ja} = \{0; 0.1; 0.2; 0.3; 0.4; 0.5; 0.6; 0.7; 0.8; 0.9\}, M(E_{ja}) = 0.3; \)
- for \( 0.4000285 < \alpha < 0.5000285 \): \( \Delta \alpha = 0.1, E_{ja} = \{0; 0.1; 0.2; 0.3; 0.4; 0.5; 0.6; 0.7; 0.8; 0.9\}, M(E_{ja}) = 0.25; \)
- for \( 0.5000285 < \alpha < 0.6000285 \): \( \Delta \alpha = 0.1, E_{ja} = \{0; 0.1; 0.2; 0.3; 0.4; 0.5; 0.6; 0.7; 0.8\}, M(E_{ja}) = 0.2; \)
- for \( 0.6000285 < \alpha < 0.7000285 \): \( \Delta \alpha = 0.1, E_{ja} = \{0; 0.1; 0.2; 0.3; 0.4; 0.5; 0.6; 0.7; 0.8\}, M(E_{ja}) = 0.15; \)
- for \( 0.7000285 < \alpha < 0.8000285 \): \( \Delta \alpha = 0.1, E_{ja} = \{0; 0.1; 0.2; 0.3; 0.4; 0.5; 0.6; 0.7; 0.8\}, M(E_{ja}) = 0.1; \)
- for \( 0.8000285 < \alpha < 0.9000285 \): \( \Delta \alpha = 0.1, E_{ja} = \{0; 0.1; 0.2; 0.3; 0.4\}, M(E_{ja}) = 0.05; \)
- for \( 0.9000285 < \alpha < 1 \): \( \Delta \alpha = 0.099715, E_{ja} = \{0; 0.1; 0.2\}, M(E_{ja}) = 0; \)

Further, we find a dot estimation of sufficiency of ecological sustainability first region as:

\[
F(\tilde{E}_1) = \frac{1}{1} \int_0^1 M(E_{ja}) d\alpha = 0.225142.
\]

By similar actions it is establish dot estimation for ecology sustainability on other levels:

- for level \( u_2 \): \( F(\tilde{E}_2) = 0.230096; \)
- for level \( u_3 \): \( F(\tilde{E}_3) = 0.292914; \)
- for level \( u_4 \): \( F(\tilde{E}_4) = 0.497489; \)
- for level \( u_5 \): \( F(\tilde{E}_5) = 1. \)

As region with most satisfactory indicator of ecological sustainability we choose region having greatest dot estimation. It is region \( u_5 \) in resulted example. On the second place – \( u_4 \), on the third – \( u_3 \), on the fourth – \( u_2 \) and, at last, on the fifth – \( u_1 \). Thus, in accepted supposes the final scales for estimation of ecological sustainability will look as it is shown on the fig.2.
After established the proved scale for estimation of ecology sustainability we will obtain a dot estimation for indicators of sustainability of air quality, water quality, land quality and biodiversity, obtained in work [2] in corresponding values form on hundred-scores system calculator: \( x_1=37.5; x_2=62.5; x_3=50; x_4=62.5 \). For obtain final estimation we will construct following trivial implicative rules:

- If sustainability of air quality (AQ) is low and sustainability of water quality (WQ) is low and sustainability of land quality (LQ) is low and sustainability of biodiversity (NB) is low, then ecology sustainability (ES) of region is low;
- If sustainability of air quality (AQ) below the average and sustainability of water quality (WQ) below the average and sustainability of land quality (LQ) below the average and sustainability of biodiversity (NB) below the average, then ecology sustainability (ES) of region below the average;
- If sustainability of air quality (AQ) average and sustainability of water quality (WQ) average and sustainability of land quality (LQ) average and sustainability of biodiversity (NB) average, then ecology sustainability (ES) of region is an average;
- If sustainability of air quality (AQ) above the average and sustainability of water quality (WQ) above the average and sustainability of land quality (LQ) above the average and sustainability of biodiversity (NB) above the average, then ecology sustainability (ES) of region are above the average;
- If sustainability of air quality (AQ) is high and sustainability of water quality (WQ) is high and sustainability of land quality (LQ) is high and sustainability of biodiversity (NB) is high, then ecology sustainability (ES) of region is high.

Having realize these rules in MATLAB/Fuzzy Sets Toolbox (fig.3), on the basis of indicators \( x_1, x_2, x_3, x_4 \) will obtain final estimation of ecology sustainability at a rate of 0.295. According to the accepted scale this value is close to average level of ecology sustainability.

References